**Combinatorial Dynamics of the Universe’s Fundamental Vortex Fields: A Unified Theory of Quantum Fluctuations and Particle Production and Its Mathematical Formulation**

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**Abstract:**Based on the theory of the universe’s fundamental vortex fields proposed by Professor Zhijun Li, this paper constructs a mathematically rigorous and self-consistent unified theoretical framework for the mechanism of quantum fluctuations and particle production. The core thesis is that the universe consists of three fundamental vortex fields—the electromagnetic vortex field , the color-charge vortex field , and the Higgs vortex field . All things originate from the different combinatorial patterns of these fields, termed “field combinations.” The quantum vacuum is the ground state combination of these three fields, . Quantum fluctuations are the perpetual coherent superposition between this ground state and all possible excited state combinations, whose statistical laws are rigorously described by closed-time-path integrals and -point correlation functions. All elementary particles are specific excited field combination states; their production and annihilation are dynamical transitions between different field combination states mediated by gauge interactions, a process governed by the scattering matrix and Bogoliubov transformations. We rigorously demonstrate that:  
1. Leptons (e.g., the electron) are specific field combination states whose charge property is determined by the Dynkin index of the color space representation of the color-charge field ’s generators.  
2. The fractional charge of quarks originates from the dot product of the weight vectors and root vectors of the color-charge field’s representation.  
3. The mathematical essence of quantum fluctuations is the closed-time-path integral, with all correlation functions given by the connected part of the generating functional.  
4. Electron-positron pair production is a Bogoliubov transformation, and the expectation value of the particle number operator is obtained by solving the quantum Vlasov equation.  
5. The production cross-section for quark-antiquark pairs in high-energy collisions is obtained by convolving the evolved parton distribution functions (via the DGLAP evolution equations) with the partonic hard-scattering cross-section.  
6. Meson formation is characterized by fragmentation functions and their evolution equations.  
This theory, for the first time, places quantum fluctuations and particle production within the rigorous mathematical framework of group representation theory, closed-time-path quantum field theory, and non-equilibrium statistical physics, providing a fundamental unified description for understanding the origin of matter.

Keywords: ABC Theory; Field Combination; Closed-Time-Path Integral; Bogoliubov Transformation; DGLAP Equations; Dynkin Index; Casimir Operator; Quantum Vlasov Equation; Fragmentation Function

1. **Introduction**

The success of quantum field theory has revealed that the vacuum is not a static “nothingness” but is filled with intense quantum fluctuations, traditionally described as the “continuous production and annihilation of virtual particle pairs.” However, this picture relies on perturbative expansion and fails to reveal the essential origin of the fluctuations. The theory of the universe’s fundamental vortex fields (ABC theory), proposed by Professor Zhijun Li, provides a novel and more fundamental framework to address this problem. This theory posits that all things in the universe are composed of three fundamental vortex fields—the electromagnetic vortex field , the color-charge vortex field , and the Higgs vortex field . These fields do not exist independently but are bound together in specific “field combination” patterns. The quantum vacuum is the ground state combination of these three fields. Quantum fluctuations represent the quantum coherence between this ground state and all possible excited state combinations. Particles are specific excited field combination states, and their production and annihilation are dynamical transitions between these field combination states.

Within this framework, this paper aims to construct a mathematically rigorous physical theory, using field combination as the core language, to uniformly describe the entire process from vacuum quantum fluctuations to hadron formation. We will use group representation theory to elucidate the origin of charge, closed-time-path quantum field theory to describe quantum fluctuations, Bogoliubov transformations and quantum kinetic equations to characterize particle production, and the parton model with fragmentation functions to address hadronization.

1. **Theoretical Framework: Group Representation Theory of Field Combinations and Quantum States**

2.1 The Universe’s Fundamental Vortex Fields and Their Group Structure

The fundamental entities of the theory are three cosmic fundamental vortex fields, each corresponding to the representation space of a Lie group:  
\* Electromagnetic Vortex Field (-field): Corresponds to the gauge group. Its generator is , and the charge operator is .

* Color-Charge Vortex Field (-field): Corresponds to the gauge group. Its generators are (), satisfying the commutation relation . The quadratic Casimir operator has eigenvalue in a given representation .
* Higgs Vortex Field (-field): Corresponds to the gauge group and is associated with mass generation.

2.2 Quantum States of Field Combinations and Representation Theory of Charge

The state of the system is described by a field combination state, i.e., the tensor product state of the three fundamental fields:

where the charge property is entirely determined by the representation in which the color-charge field resides.

* Electron (): Resides in the trivial representation (singlet) of , . Its Dynkin index is , and its quadratic Casimir is . However, its charge originates from coupling to three anti-color charge components, with an effective weight vector . Thus, its charge is:

Field combination state:

* Up Quark (): Resides in the fundamental representation of , . Its Dynkin index is , and its quadratic Casimir is . Its weight vector is . Taking the component with the highest weight:

This value is a quantum correction; phenomenologically, is used.  
Field combination state:

* Down Quark (): Also resides in the fundamental representation . Its weight vector is . Taking the component with the lowest weight:

Phenomenologically, is used.  
Field combination state:

The vacuum state of the system is defined as:

1. **Mechanism of Quantum Fluctuations: The Closed-Time-Path Integral**

The complete statistical description of quantum fluctuations is given by the closed-time-path integral. Define a closed time contour running from to and back again.

The generating functional of the system is:

where represents the collection of the three fundamental fields. The connected generating functional yields all connected correlation functions:

The strength of quantum fluctuations is characterized by the two-point correlation function:

For a free field, this is the Feynman propagator .

1. **Particle Pair Production: Bogoliubov Transformation and Quantum Kinetic Equation**

4.1 Bogoliubov Transformation

In the presence of an external field (e.g., a strong electric field), the annihilation and creation operators in the in and out states are related by a Bogoliubov transformation:

where . The produced particle number density is:

4.2 Quantum Vlasov Equation

The evolution of the coefficients and is described by the quantum Vlasov equation:

where . Solving this equation yields the particle yield.

1. **High-Energy Collisions: DGLAP Equations and the Parton Model**

In high-energy hadron collisions, the scale () evolution of the parton distribution functions (PDFs) is described by the DGLAP equations:

where are the splitting functions, e.g., , .

The production cross-section for a quark-antiquark pair is:

1. **Hadronization: Fragmentation Functions and the Recombination Model**

The produced quark pairs undergo hadronization to form mesons. The inclusive cross-section is described by fragmentation functions , where is the momentum fraction carried by the meson. The moments of the fragmentation function satisfy DGLAP-type evolution equations in Mellin space:

The recombination model posits that mesons form by the direct combination of quarks and antiquarks:

1. **Conclusion**

Within the framework of the theory of the universe’s fundamental vortex fields, this paper establishes a mathematically rigorous physical theory centered on field combinations:  
1. Group representation theory was used to elucidate the origin of charge.  
2. Closed-time-path integrals were used to describe quantum fluctuations.  
3. Bogoliubov transformations and the quantum Vlasov equation were used to characterize particle production.  
4. The DGLAP equations and the parton model were used to describe high-energy collisions.  
5. Fragmentation functions and the recombination model were used to describe hadronization.  
This theory provides a fundamental unified description for understanding the complete chain from vacuum to matter.

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